Test 2 Review Answers

1) Find y' if $y = \dfrac{xe^{-x}\cos x}{\ln x}$

Ans:

$$y' = \frac{e^{-x}([(1-x)\cos x - x\sin x]\ln x - \cos x)}{(\ln x)^2}$$

2) Find the x-coordinate(s) when the given function has a horizontal tangent line

$$T(x) = x^2 e^{1-3x}$$

Ans: $x = 0$ and $x = 2/3$

3) Find $\frac{dy}{dx}$ if $y = \sqrt[3]{x + \sqrt{2\sec x}}$

Ans:

$$y' = \frac{1}{3}[x + (2\sec x)^{1/2}]^{-2/3}(1 + \frac{1}{2}(2\sec x)^{-1/2}\sec x\tan x)$$

or

$$y' = \frac{1}{3}[x + (2\sec x)^{1/2}]^{-2/3}(1 + \frac{1}{2\sqrt{2}}(\sec x)^{1/2}\tan x)$$

4) Find y' if $\ln(xy) = e^{2x}$

Ans: $y' = 2ye^{2x} - \dfrac{y}{x}$

5) Use logarithmic differentiation to find

$$\frac{d}{dx}(x^{\ln\sqrt{x}})$$

Ans: $x^{\ln\sqrt{x}} \cdot \dfrac{\ln x}{x}$ or $(\ln x)x^{(\ln\sqrt{x})-1}$

6) Calculate $\dfrac{d}{dx}\tan(\arctan x)$ two different ways- First take the derivative and then simplify your answer. Next, simplify the expression first and then take the derivative. (Hint: $\cos(\arctan x) = \frac{1}{\sqrt{1+x^2}}$....do you know how to prove it?)

Ans: You should get 1 both ways that you compute it.

7) In a healthy person of height x inches, the average pulse rate in beats/minute is given by

$$P(x) = \frac{596}{\sqrt{x}}$$

Use differentials to estimate the change in pulse rate that corresponds to a height change from 49 to 50 inches.

Ans: $dP = \frac{-298}{343}$ beats per minute.

8) Given the function $f(x) = \ln(2 - x)$, (a) find the linearization $L(x)$ at $a = 1$, (b) use $L(x)$ to approximate $\ln(1.1)$, and (c) calculate $\ln(1.1)$ on your calculator. How close is your approximation?

Ans: (a) $L(x) = 1 - x$

(b) $\ln 1.1 \approx 0.1$

(c) $\ln 1.1 = 0.0953$